Wittgenstein's Quasi-Intuitionism

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Abstract

Is Wittgenstein an intuitionist? It's unclear whether he rejects or emends Brouwer. His logical atomism relies on correspondence, while his mathematical constructivism doesn't. Scholars are divided. Following Russell, Wittgenstein endorses a *fact-based version of correspondence*. The Aristotelian truth-definition, which can be reduced to "*x* is true iff *x* corresponds to some fact", is restricted to a subclass of truth-bearers, namely *elementary* propositions whose truth consists in their correspondence to state of affairs. On the other hand, Wittgenstein dismisses *the law of excluded middle*, "$(x)Fx \vee (\exists x){\sim}Fx$". "$P \vee {\sim}P$", for instance, doesn't hold for infinite sequences since it doesn't tell whether the pattern φ (any particular arrangements of digits) occurs in the infinite expansion π or not. In this paper, I shall examine the tension between *realism* and *intuitionism* in Wittgenstein's philosophy, where his relying on correspondence seems to conflict with his rejection of the law of excluded middle. I shall finally accommodate the two within a single, coherent view on mathematics that might be seen as *quasi-intuitionism*, where mathematics is reduced to mental manipulations of signs (consistently with any degree of constructivism) that yet resist any mental dependency.

Legend has it that Wittgenstein, along with many members of the Vienna Circle, was present at Brouwer's lecture "Mathematik, Wissenschaft und Sprache" in Vienna in 1928, and that subsequently he resumed his philosophy with an increasing attention to the foundations of mathematics. Like Hacker, some scholars view "Wittgenstein's later philosophy as a generalized intuitionist theory" (1972: 104). Others disagree. Wittgenstein, argues Hintikka, never considered mathematics "to be a matter of intuitive mental constructions" (1996: 81f.). Yet others occupy a middle ground. Marion (2003), for instance, believes that Wittgenstein shares the basics of Brouwer's intuitionism already in the *Tractatus* (1922). Perhaps, suggests Marion, Wittgenstein's notion of operation simply tries to address the main issue of Brouwer's basic intuition (*Urintuition*), namely how one can obtain numbers by repeating an initial intuition. After all, his introduction to the calculus of the truth-functions as based on truth-operations and his definition of natural numbers as exponents of operations (*TLP* 5.234–5.2341/6.021) might well serve this purpose.

In *Philosophical Investigations* (1953), however, the later Wittgenstein labels mathematical intuition an *unnecessary shuffle*. It's unclear whether he intends to emend or dismiss the notion, but he doesn't turn to Platonism either. To the contrary, his rejection is straightforward. In fact, Wittgenstein's view of mathematics is not descriptive, although his issue with Platonism is not with the existence of mathematical objects. After all, constructivists don't reject it either. It rather pertains to the objectivity of mathematical truths, which intuitionists reduce to fiction. Yet, Brouwer's emphasis on *free creativity* doesn't mean 'anything goes'. It simply points to a different kind of objectivity whose commitments still require that "a meaning of a statement be fixed by determining the circumstances under which it is true or false" (Wright 1980: 9). Endorsing this notion doesn't commit to Platonism yet. A Platonist views these truth-conditions of meaning as defined by a strong correspondence theory of truth, but Wittgenstein does not. Furthermore, this view implies the existence of objective truths that lack a method of verification, which Wittgenstein clearly condemns.

In this paper, I shall examine the tension between *realism* and *intuitionism* in Wittgenstein's philosophy, where his relying on correspondence seems to conflict with his rejection of the law of excluded middle. I shall finally accommodate the two within a single, coherent view of mathematics that might be seen as *quasi-intuitionism*.

1) Language and Correspondence

In the *Tractatus*, Wittgenstein's philosophy questions the symbolic relation between words and things that pertains to any language. The main issue of this symbolism concerns the conditions of both "sense" (provided by the syntax) and "symbolic reference" (meaning). "A logically perfect language has rules of syntax which prevent nonsense, and has single symbols which always have a definite and unique meaning" (*TLP*, x), comments Russell. This language remains detached from reality and doesn't properly exist. Nevertheless, it represents an ideal model that works by default.

By means of the language we can assert or deny facts. In Wittgenstein's view, the entire business of the language consists in its assertive relation to facts. He grounds this relation on a metaphysical correspondence that Russell calls "structural".

> In order that a certain sentence should assert a certain fact there must, however the language may be constructed, be something in common between the structure of the sentence and the structure of the fact (*TLP*, x–xi).

The thesis of Wittgenstein – "perhaps the most fundamental" (ibid.) says Russell – slightly but significantly differs from the classic correspondence theory of truth, though. According to the Aristotelian ομοιωσις (*likeness*) truth is a relational property (*adaequatio* or *conformitas*) and involves a characteristic relation to some portion of reality. His definition can be reduced to:

(**Def-1**) *x* is true iff *x* corresponds to some fact;
x is false iff *x* does not correspond to any fact.

Following Aristotle, many variations on correspondence have redefined the theory. Wittgenstein follows Russell and endorses a fact-based version of correspondence.

> Thus a belief, claims Russell, is true when there is a corresponding fact, and is false when there is no corresponding fact (1912: 129).

This account of truth supports their program of logical atomism. Central to this latter is a certain analysis of language whose main claims rethink the interplay of names, symbols, and facts in the following way.

> (**a**) A *name* (*sign*) is a *simple symbol* that has no parts that are themselves symbols.

> I indicate them [names], says Wittgenstein, by single letters ('*x*', '*y*', '*z*'). I write elementary propositions as functions of names, so that they have the form '*fx*', '$\phi(x,y)$', etc. Or I indicate them by the letters '*p*', '*q*', '*r*' (*TLP*, 4.24).

Sign and symbol differ from each other. The former is an element of syntax that usually consists of letters or strings of letters, but it also includes gestures, sounds, diagrams, etc.

> A *symbol* (or *expression*), clarifies Potter, is what a sign becomes when it is read as a linguistic item in a particular way (2000: 164).

Therefore, symbols rely on perceptible signs, including propositional signs (propositions) and signify in manifold ways. And symbols refer to types while signs don't.

> Mere signs do not have types since the type is a function of how it symbolizes, i.e. of the symbol, not the sign. If, on the other hand, we can talk of symbols rather than signs, they already have types which we are powerless to change (2000: 171).

(**b**) The world consists of *facts*, which are always complex. A fact, says Wittgenstein, "is the existence of *states of affairs*" (*Sachverhalt*) (*TLP*, 2), while a fact that consists of two or more facts is a *Tatsache* (just fact). A *state of affair*, explains Russell, "although it contains no parts that are facts, nevertheless does contain parts" (*TLP*, xiv), though. It has no factual parts but is a complex of parts (things, objects) anyway. And we can in theory even know its constituents (objects) because, after all, any complex logically presupposes simples. Properly stated, "a state of affairs (a state of things) is a combination of objects (things, *Sachen, Dingen*)" (*TLP*, 2.01), which are always "simple" (*einfach*) (*TLP*, 2.02). In this sense, "the world is the totality of facts" (*TLP*, 1.1).

These assumptions underpin Russell-Wittgenstein's *logical atomism*. The basic truth-definition (Def-1) can now be restricted to a subclass of truth-bearers, namely *elementary* or *atomic* propositions whose truth consists in their correspondence to *state of affairs* (Wittgenstein) or *atomic facts* (Russell). Roughly put:

> (**Def-2**) if *x* is elementary, then *x* is true iff *x* corresponds to the existence of some state of affairs (where *x* stands for proposition).

The restricted definition (Def-2) offers a default model for non-elementary truth-bearers, whose truth-values can be recursively explained in terms of the logical structure of their simpler components. For example, a sentence of the form 'not-*p*' is true iff '*p*' is false; a sentence of the form '*p*•*q*' is true iff '*p*' is true and '*q*' is true; a sentence of the form '$p \vee q$' is true iff '*p*' is true or '*q*' is true, etc. These recursive truth conditions can be reapplied until the truth of a non-elementary claim of arbitrary complexity is reduced to the truth (or falsehood) of its elementary, atomic constituents.

B-assumption and Def-2 lead to Wittgenstein's conception of language. "A proposition (true or false), explains Russell, asserting an atomic fact is called an atomic proposition" (*TLP*, xv). Since all atomic claims are logically independent of each other, logical inferences are only concerned with not-atomic claims, namely molecular claims.

2) The Law of Excluded Middle

In propositional logic, classic change of quantifier rules state four logical equivalences:

$(x)Fx :: \sim(\exists x)\sim Fx$
$\sim(x)Fx :: (\exists x)\sim Fx$
$(\exists x)Fx :: \sim(x)\sim Fx$
$\sim(\exists x)Fx :: (x)\sim Fx$

From them we can also derive a valid disjunction also known as *the law of excluded middle* (henceforth LEM): $(x)Fx \vee (\exists x)\sim Fx$ (everything is *a* or something isn't *a*).

LEM is a logical principle consistent with a finite group of objects, usually intended (and Wittgenstein certainly does) as parts of the spatio-temporal world. As soon as mathematics turns to infinity, hold intuitionists, LEM becomes illegitimate, though. Wittgenstein agrees with Brouwer's counterexample to it, namely the pendulum case whose binary oscillatory shrinking number is neither rational nor irrational, in violation of LEM. Nevertheless, argues Marion, he might disagree with Brouwer's reasons.

> If mathematics [clarifies Wittgenstein] was the investigation of empirically given aggregates, one could use the exclusion of a part to describe what was not excluded and in that case the non-excluded part wouldn't be equivalent to the exclusion of the others (1994: 156).

He rejects the idea that if a proposition is valid for one region of mathematics it's not necessarily valid for a second region as well. On the contrary, "the applicability of logic requires that it is a priori possible to tell if the proposition is true or false" (Marion, 2003: 120). In other words, Wittgenstein is advocating for an unrestricted rejection of LEM in logic. His main reasons pertain to the nature of mathematics, which is *modal* and *constructive*.

> When the classical mathematician, argues Fogelin, attempts to establish '*P*' by assuming "~*P*," he is misapplying the rule of indirect proof by assuming the contrary – not the contradictory – of the proposition to be established (1968: 269).

In his proof, the correct assumption ~*Rp* (for a certain *rule R* there is a *proposition p* for which *R* doesn't apply) is replaced by *R~p*, namely ~*P*.

The contrary destroys the *modal character* of mathematical propositions and blends them with the empirical. In fact, Wittgenstein's mathematical constructions aren't *descriptive* but *prescriptive* (normative) propositions – Kripke holds the same view: before the addition 68+5, "the relation of meaning and intention to future action is *normative, not descriptive*" (1982: 37). In this way, "mathematics forms a network of norms" (*RFM*,1956: V, 46). A proposition proved by means of a proof serves as a rule. Acknowledging '5×5=25' as a law turns it into 'it's a law that 5×5=25'. Like in any indirect proof, LEM's acknowledgment of ~*P* lays down a new rule abruptly and thus becomes "an engine of creation and not, as the Platonists believe, a device for discovering new mathematical facts" (Fogelin, 1968: 270).

Furthermore, LEM, $P \vee \sim P$, doesn't hold for infinite sequences. It doesn't tell, for instance, whether the pattern φ

(any particular arrangements of digits) occurs in the infinite expansion π or not (*PI*, 1953: 119[e]).

> When someone sets up the law of excluded middle, he is as it were putting two pictures before us to choose from, and saying that one must correspond to the fact. But what if it is questionable whether the pictures can be applied here? And if you say that the infinite expansion must contain the pattern φ or not contain it, you are so to speak shewing us the picture of an unsurveyable series reaching into distance. But what if the picture began to flicker in the far distance? (*RFM*, IV, 10).

"In itself, a proposition is neither probable nor improbable. Either an event occurs or it does not: there is no middle way" (*TLP*, 5.153). But mathematics differs from philosophy in this regard: mathematics isn't about reality but forms (formal properties) of reality, namely rules for relating portions of reality.

Before a mathematical sentence of a completely formalized theory, explains Dummett (1959), a Platonist will hold that there exist either a proof or a disproof of that statement, following the law of excluded middle. A true statement entails the existence of such a proof even though it isn't discovered yet. But for Wittgenstein, all the same there exists no proof yet. In his view, the sentence just remains undecided: *either… or* is replaced with *intention*. It's up to us whether to accept the proof and adopt a new rule of language. After choosing the initial conventions, we have no further business with the proof. We can only follow it. And this Wittgenstein calls *the rule-following argument*.

3) Conclusions

Language and world have *something in common* that is expressed in terms of *symbolic relation*. Wittgenstein accordingly redefines the correspondence theory of truth. His leading assumption is that a common part must be shared by sentence and fact at a structural level, although this part cannot be said but only shown in language. He postulates a correlation between the metaphysical level of the world, represented by *things and* (atomic or complex) *facts*, and its semantic level where we understand *physical signs* as *meaningful symbols*. Then he offers a *fact-based version of correspondence*, which I read as a plausible variation of the *finite universe method*.

However, mathematics and world *have no common part*. Their correspondence just fails. Mathematical propositions can only pretend to be propositions (*Scheinsätze*) but truly are something else. "The propositions of mathematics are equations, and therefore pseudo-propositions" (*TLP*, 6.2), namely propositions that do "not express a thought" (*TLP*, 6.21). There are "no objects", explains Marion, "for which arithmetical terms go proxy and *a fortiori* no connections between such putative objects into facts that would be pictured in mathematical propositions" (111). Mathematics, per se, is meaningless, namely without reference. In order to have meaning, the mathematical discourse must refer to the empirical discourse. "Concepts occurring in 'necessary' propositions must also have a meaning in non-necessary ones" (*RFM*, IV–41), namely "it's essential to mathematics that its signs are also employed in *mufti*" (*RFM*, IV–2). "The numerical '2'", comments Fogelin, "occurs in the empirical proposition "There are 2 horses on the elevator" and also in the necessary proposition "2+2=4" … without *significant occurrences* in expressions of the first sort, the numerical '2' could not have *significant occurrences* in expressions of the second sort" (1968: 271).

Clearly, this sounds anti-Platonic.

> Still there's something inherently compelling [says Fogelin] about the following reasoning: '5×5=25' expresses a true proposition. Thus, there must exist a domain of objects that it is true of. Furthermore, it expresses a necessarily true proposition; hence these objects must be ideal, not empirical, objects (1968: 267).

I believe that Wittgenstein identifies this quasi-Platonic (Wright, 1980: 167–181) domain with tautological structures, namely equations. In this sense, mathematics rests on the quasi-intuitive notion of operation: "The logic of the world, which is shown in tautologies by the propositions of logic, is shown in equations by mathematics" (*TLP*, 6.22). Roughly put, mathematics is all about operations for carrying out tautologies. Consistently, the proof of an equation corresponds to the method of tautology" whose aim is "to make evident the agreement between two structures (Marion, 2003: 113) – i.e., Fogelin's domain.

In Wittgenstein's eyes, Brouwer's basic intuition isn't psychological but "a primitive sign" and "an element of a calculus" (*PG*, 1974: 322). Thus mathematics is reduced to a mental manipulation of signs (consistently with any intuitionism and constructivism) that yet resist any mental dependency. These signs have a sort of *independent life* that Wittgenstein believed be lacking in Brouwer. This I call "quasi-intuitionism", which Wittgenstein's radical rejection of LEM corroborates.

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